TM-956
1680

Fermilab

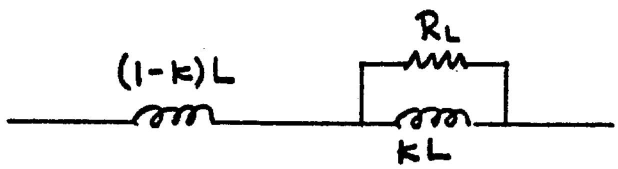
QUENCH PROTECTION CIRCUIT FOR SUPERCONDUCTING MAGNETS WITH EDDY CURRENTS

R.E. Shafer

April 2, 1980

In order to protect superconducting magnets from being damaged by quenches, a quench protection monitoring circuit is used to monitor the voltage waveform generated across the terminals of a superconducting magnet when it is being driven by a current waveform. This voltage waveform is compared to the expected waveform, and if a discrepancy is observed due to a portion of the superconductor going normal, the power supply is shut down. As superconducting magnets have eddy current losses which cause the voltage waveform to deviate from an ideal inductance, the waveform generated in the protection circuit must be modified also. The purpose of this note is to discuss the modification of the protection circuit to compensate for eddy current losses. A specific example will be given.

In a magnet with eddy current, some fraction k of the total inductance is strongly coupled to a resistive eddy current loop with an effective resistance R_L. If L is the dc inductance, then the magnet looks like the following for a simple eddy current loss (See Appendix):



If we define $\tau = kL/R_L$ then we may write the impedance as

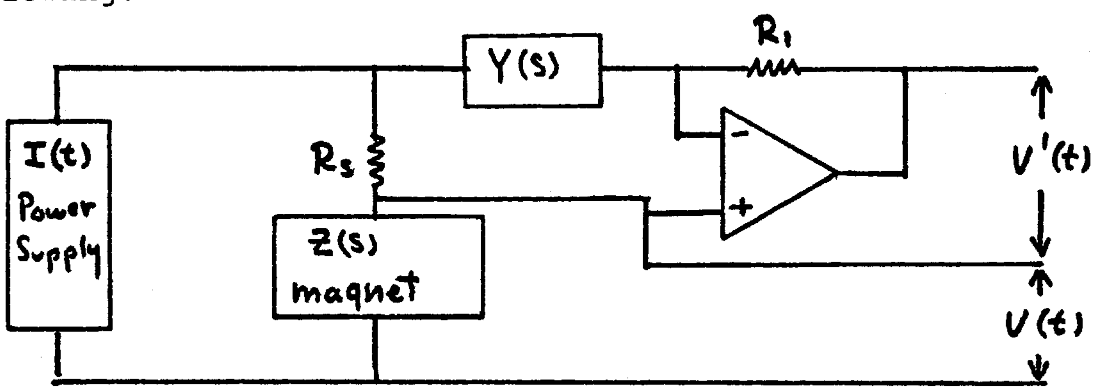
$$Z(s) = (1-k)Ls + \frac{kLs}{1+\tau s} \; ; \quad s = \text{Laplace operator} \qquad [1]$$
$$= L\,G(s)$$

Hence if the magnet is energized with a current waveform I(t) the voltage waveform V(t) generated across its terminals is the transform of

$$V(s) = Z(s)\, I(s) = L\, G(s)\, I(s) \qquad [2]$$

where I(s) is the transform of I(t).

We would like to generate a voltage waveform similar to V(t) using a differentiator circuit. Let us consider specifically the following:



In this circuit R_s is shown as a series shunt, although the actual voltage signal monitoring the current waveform may be derived internally in the power supply.

In the above circuit $V'(s) = R_1\ Y(s)\ R_s\ I(s)$ [3]

and $V(s) = Z(s)\ I(s) = L\ G(s)\ I(s)$

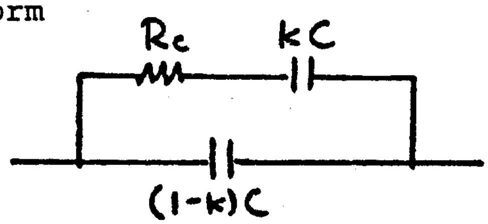
Hence in order that V(s) = V'(s)

$$L\, G(s) = Y(s)\, R_1 R_s \qquad [4]$$

to satisfy this requirement Y(s) must be of the form $\left(C = \frac{L}{R_1 R_s} \right)$:

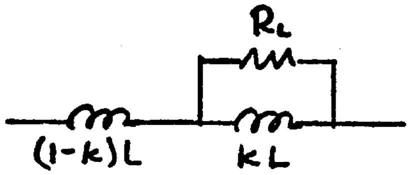
$$Y(s) = \frac{L}{R_1 R_s}\, G(s) = (1-k)\, Cs + \frac{kCs}{1+\tau s} \qquad [5]$$

This is a circuit of the form



where $\tau = kC/R_c$

As a specific example let's consider a 22' doubler dipole. Its equivalent circuit for eddy current losses is approximately



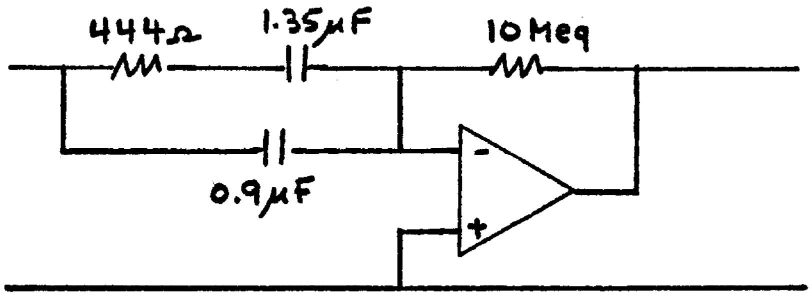
$L = 45\,mH$

$k = 0.6$

$R_L = 45\,\Omega$

We will use R_s = .002 ohms. That is to say, a ±10 volt signal represents ±5000 amps. If R_1 is chosen to be 10 Meg, then C = 2.25 μF. Since k = .6, then $R_c = L/R_LC = 444\,\Omega$.

Hence the differentiator circuit becomes



If the equivalent circuit model for the magnet is not known, then it can be easily determined as follows:

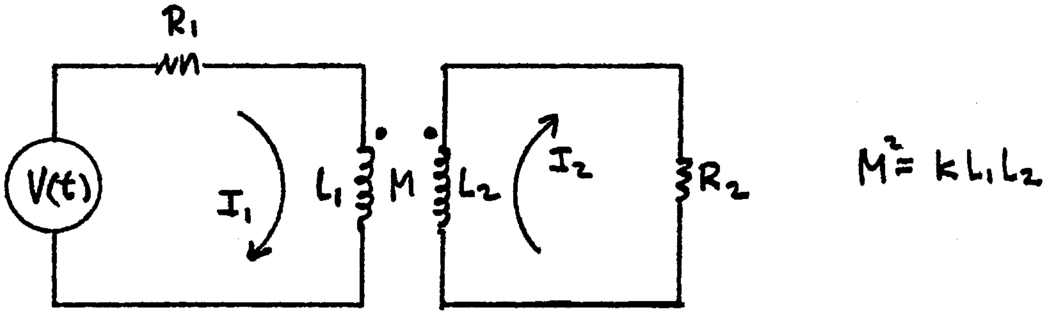
1. Build the differentiator circuit assuming an ideal magnet of known inductance L.
2. Compare signals V'(t) and V(t), observing especially the spikes produced in V'(t) - V(t) when $d\frac{I(t)}{dt}$ is suddenly changed from one constant value to another, causing V(t) to change from a dc value V_1 to a new value V_2.
3. Measure amplitude and time constant of spike.
4. $k = V_{spike}/(V_2-V_1)$
5. $\tau = \tau_{spike}/(1-k)$

The model for Y(s) may be expanded to include additional eddy current loops by paralleling additional kR_cC circuits. If the superconducting magnet leads are resistive, then a shunt resistance across Y(s) will compensate for that.

In the actual implementation of this differentiation circuit, one must of course take into account the finite gain-bandwidth limitations of op-amps etc. Hence in the above example the 10 meg feedback resistor would have to be replaced with a much smaller value (say 100k) and the circuit followed by a dc gain of 100. The point of this TM was not to discuss the actual implementation of the protection circuit but to discuss the proper differentiation circuit configuration required.

Appendix: Equivalent Circuit for Eddy Current Loss in Magnet

We consider the following model for a simple eddy current loss in a magnet. The primary has a DC inductance L_1 and series resistance R_1. The primary inductance is loosely coupled via flux linkage to a secondary of inductance L_2 and R_2. This secondary may be a conducting structure linking a magnet yoke, or a conducting plate intercepting flux lines. It is a simplified model only in that the secondary loop has a specific L_2/R_2 time constant. Note that $M^2 = kL_1L_2$, not $k^2L_1L_2$, for reasons which will become apparent.



We consider a sinewave exitation of the form $V(t) = V_1 \sin \omega t$. The loop equations then may be written:

$$I_1R_1 + j\omega L_1 I_1 + j\omega M I_2 = V_1$$
$$I_2R_2 + j\omega L_2 I_2 + j\omega M I_1 = 0 \qquad [A-1]$$

Eliminating I_2 from the first equation we have

$$I_1R_1 + j\omega L_1 I_1 + j\omega M\left[\frac{-j\omega M}{R_2 + j\omega L_2}\right] I_1 = V_1 \qquad [A-2]$$

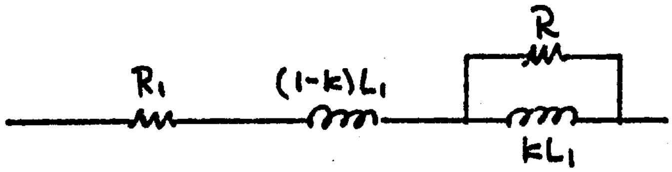
The impedance as seen from the primary is then, using $\tau = L_2/R_2$ and $M^2 = kL_1L_2$:

$$Z_1 = \frac{V_1}{I_1} = R_1 + j\omega L_1 + \frac{\omega^2 k L_1 \tau}{1 + j\omega\tau}$$

$$= R_1 + \frac{\omega^2 k L_1 \tau}{1+(\omega\tau)^2} + j\omega\left[L_1 - \frac{k(\omega\tau)^2 L_1}{1+(\omega\tau)^2}\right]$$

$$= R_1 + \frac{\omega^2 k L_1 \tau}{1+(\omega\tau)^2} + j\omega\left[(1-k)L_1 + \frac{kL_1}{1+(\omega\tau)^2}\right] \qquad [A-3]$$

This is recognized as the equation for the following equivalent circuit:



where R is defined as kL_1/τ. The above equation may then be rewritten as:

$$Z_1 = R_1 + \frac{(\omega\tau)^2 R}{1+(\omega\tau)^2} + j\omega\left[(1-k)L_1 + \frac{kL_1}{1+(\omega\tau)^2}\right] \qquad [A-4]$$

Asymtotic values of this equation are

Limit	Re Z_1	Im Z_1/ω
$\omega\tau \ll 1$	$R_1 + (\omega\tau)^2 R$ *	L_1
$\omega\tau \gg 1$	$R_1 + R$	$(1-k)L_1$

*The ω^2 dependence of Re Z_1 is easily observed for $\omega\tau \ll 1$, especially in superconducting magnets where $R_1 = 0$. This is the term which leads to ac power losses when superconducting magnets are ramped. The size and frequency dependence of this term is completely determined by knowledge of the frequency dependence of Im Z_1 (i.e. the inductance), or by measurement of transients during ramping.